Quantumness of cosmological perturbations

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A. Micheli and P. Peter, arXiv:2211.00182 in Handbook of Quantum Gravity

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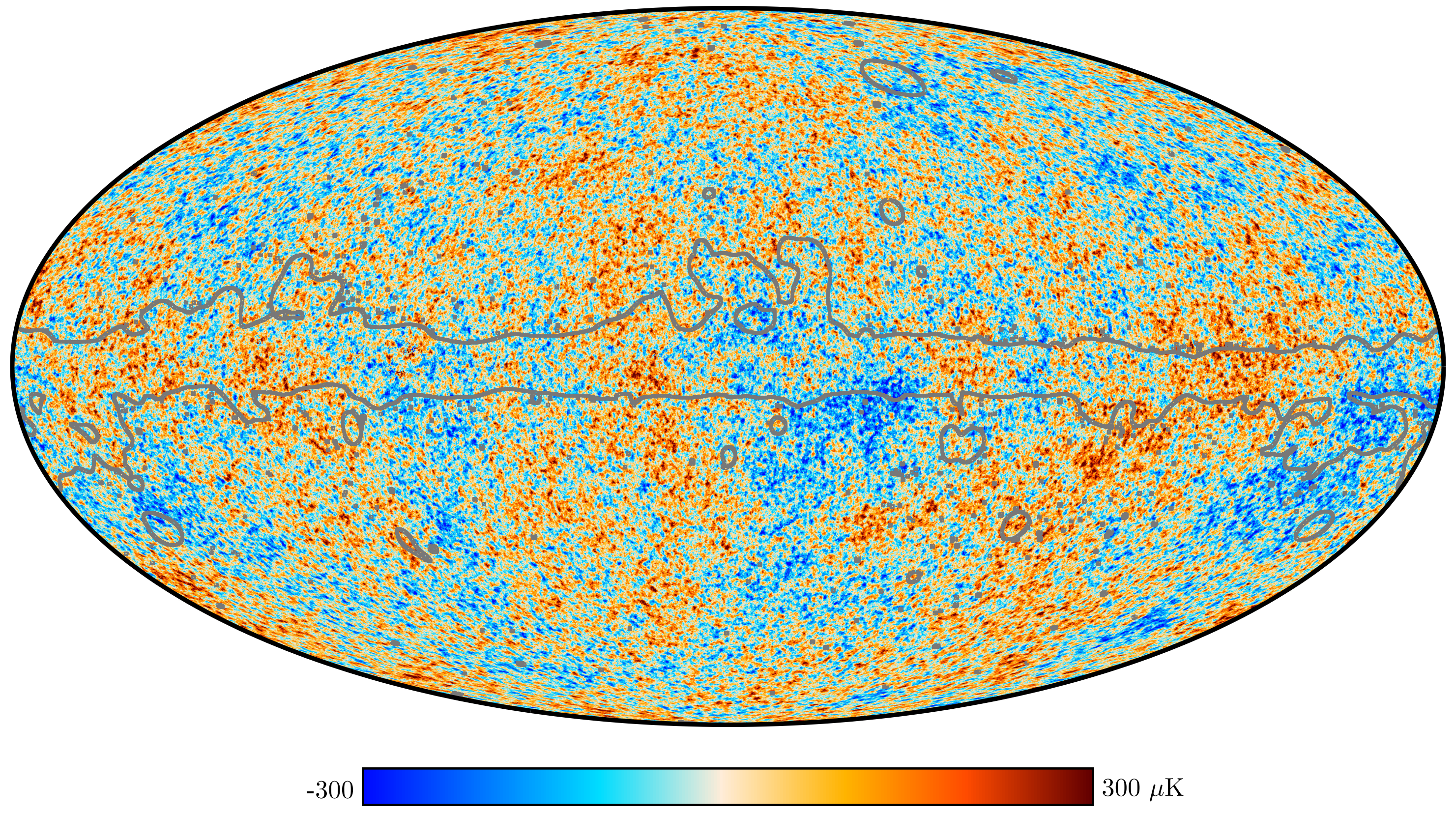
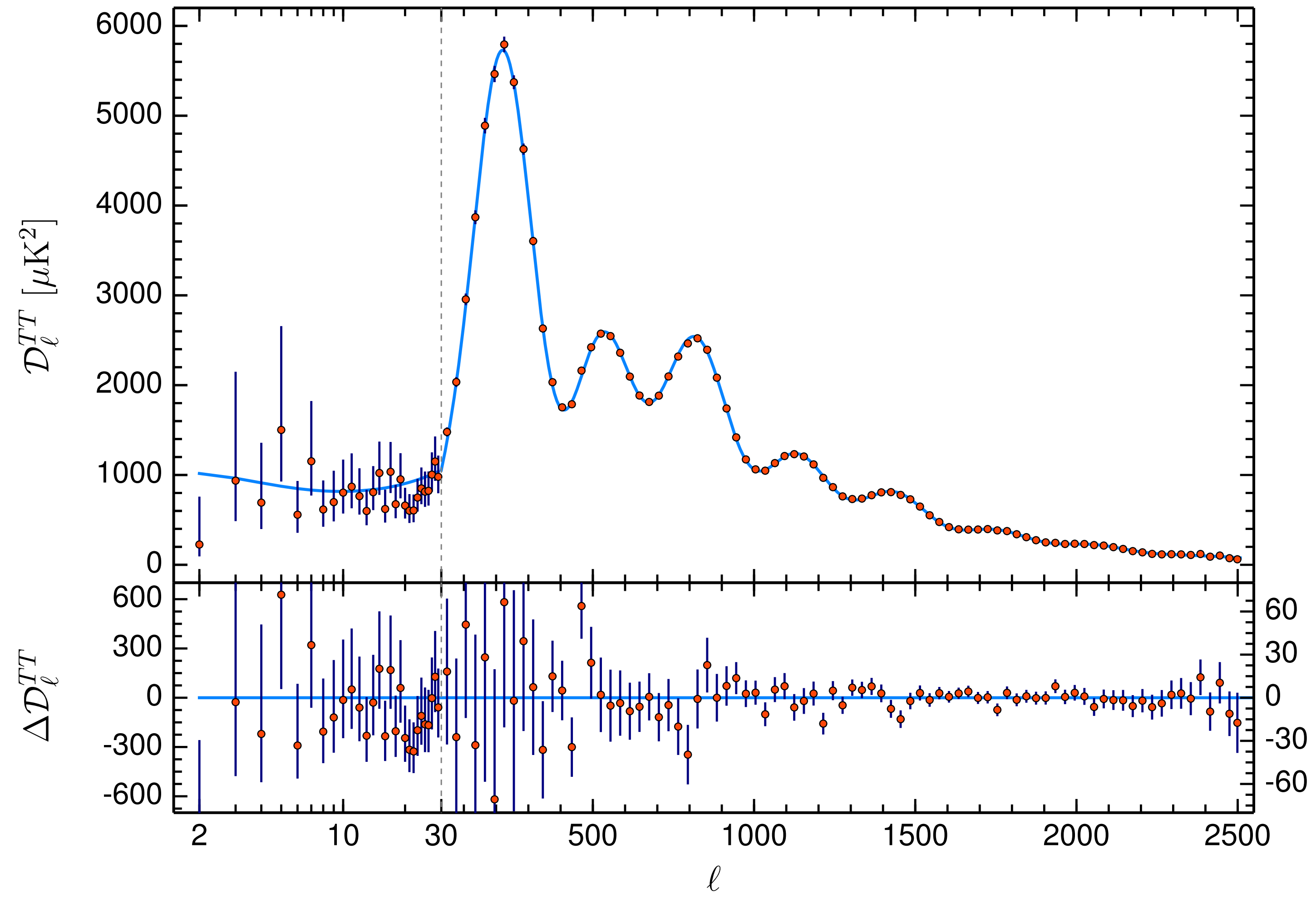
Inhomogeneities in the early Universe - I

- Primordial inhomogeneities come from (vacuum) quantum fluctuations at the beginning of inflation stretched to cosmological scales by expansion.





Questions / Motivations

- Direct proof that initial fluctuations cannot be classical? Would establish the need to quantised gravity.
- If quantum then and classical now, how did the transition happen? Quantum-to-classical transition problem.

- Indirect proof : very good agreement with observational data[1].

1. [Planck-Collaboration et al., 2020]

Plan

I. Quantum description of the state of the perturbations

II. Quantum signatures

III. Decoherence: Destruction of quantum signatures

I - Quantum state of perturbations

Classical cosmological perturbations in inflation

- GR $g_{\mu\nu}$ with a single scalar field ϕ: $$S = \int \mathrm{d}^4x \sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right]$$
- Background: Inflation, slowly rolling homogeneous $\phi_0(t)$ leading to a FLRW metric with an accelerated expansion $\ddot{a} > 0$.
- Focus on the scalar perturbations, represented by Mukhanov-Sasaki v, at linear order
- Lagrangian reads $$L = \frac{1}{2}\int \mathrm{d}^3\mathbf{k}\, L_{\mathbf{k},-\mathbf{k}} = \frac{1}{2}\int \mathrm{d}^3\mathbf{k}\left[\left(v_{\mathbf{k}}'^* - \frac{z'}{z}v_{\mathbf{k}}^*\right)\left(v_{\mathbf{k}}' - \frac{z'}{z}v_{\mathbf{k}}\right) - k^2 v_{\mathbf{k}}^* v_{\mathbf{k}}\right]$$
  where $z = M_{\mathrm{Pl}} a\sqrt{2\epsilon_1}$, $\epsilon_1 = -\dot{H}/H^2$ and $v_k^* = v_{-k}$
- Independent $\pm\mathbf{k}$ pairs, collection of parametric oscillators: $$v_{\pm\mathbf{k}}'' + \left(k^2 - \frac{z''}{z}\right)v_{\pm\mathbf{k}} = 0$$
- Perturbation amplified when $k^2 \gg z^{-1}z''$ and in slow-roll corresponds to $k(aH)^{-1} \gg 1$ i.e. mode super-Hubble

Quantum state of perturbations

- Quantisation: Conjugated field $\hat{\pi}_{\pm\mathbf{k}} = \hat{v}'_{\pm\mathbf{k}} - \frac{z'}{z}\hat{v}_{\pm\mathbf{k}}$ and $\left[\hat{v}_{\mathbf{k}}, \hat{\pi}_{\mathbf{k}'}\right] = \hbar\delta\left(\mathbf{k}+\mathbf{k}'\right)$
- Go to Hamiltonian $\hat{H}_{\mathbf{k},-\mathbf{k}} = \hat{\pi}_{\mathbf{k}}\hat{\pi}_{-\mathbf{k}} + k^2\hat{v}_{\mathbf{k}}\hat{v}_{-\mathbf{k}} + \frac{z'}{z}\left(\hat{\pi}_{\mathbf{k}}\hat{v}_{-\mathbf{k}} + \hat{v}_{\mathbf{k}}\hat{\pi}_{-\mathbf{k}}\right)$
- Harmonic oscillator + interaction with a classical source: typical QFT in curved spacetime effect e.g. Schwinger effect, Dynamical Casimir effect.
- Vacuum initial state is Gaussian + quadratic hamiltonian: state remains Gaussian
- Wavefunction in v-representation

$$\Psi_{\mathbf{k},-\mathbf{k}} = \sqrt{\frac{k}{\pi\hbar\gamma_{11}}}e^{-\frac{k}{\hbar}\frac{(1-i\gamma_{12})}{\gamma_{11}}v_{\mathbf{k}}v_{-\mathbf{k}}}$$ where $$\gamma_{11} = 2k\left\langle\hat{v}_{\mathbf{k}}\hat{v}^{\dagger}_{\mathbf{k}}\right\rangle$$ $$\gamma_{12} = \gamma_{21} = \left\langle\hat{v}_{\mathbf{k}}\hat{\pi}^{\dagger}_{\mathbf{k}} + \hat{\pi}_{\mathbf{k}}\hat{v}^{\dagger}_{\mathbf{k}}\right\rangle$$

  2-mode squeezed state
- Introduce Hermitian operators $\hat{v}^{\mathrm{r/i}}_{\mathbf{k}} = 2^{-1/2}\left(\hat{v}_{\mathbf{k}} \pm \hat{v}^{\dagger}_{\mathbf{k}}\right)$ and similarly for $\hat{\pi}^{\mathrm{r/i}}_{\mathbf{k}}$

$$\Psi_{\mathbf{k},-\mathbf{k}} = \sqrt{\frac{k}{\pi\hbar\gamma_{11}}}e^{-\frac{k}{2\hbar}\frac{(1-i\gamma_{12})}{\gamma_{11}}\left[\left(v^{\mathrm{r}}\right)^2 + \left(v^{\mathrm{i}}\right)^2\right]} = \Psi\left(v^{\mathrm{r}}\right)\Psi\left(v^{\mathrm{i}}\right)$$

NB: Not real and Imaginary part

Phase-space representation of state - I

- Density matrix $\hat{\rho} = \left|\Psi\right\rangle\left\langle\Psi\right| = \left|\Psi_{\mathrm{r}}\right\rangle\left\langle\Psi_{\mathrm{r}}\right| \otimes \left|\Psi_{\mathrm{i}}\right\rangle\left\langle\Psi_{\mathrm{i}}\right| = \hat{\rho}_{\mathrm{r}} \otimes \hat{\rho}_{\mathrm{i}}$ focus on $\hat{\rho}_{\mathrm{r}}$

- Weyl transform maps operators $\hat{O}$ to phase-space functions $O\left(v^{\mathrm{r}}, \pi^{\mathrm{r}}\right)$

$$\mathscr{W}\left(\hat{O}\right)\left(v^{\mathrm{r}}, \pi^{\mathrm{r}}\right) = \frac{1}{\hbar}\int \mathrm{d}x\, e^{-i\frac{\pi^{\mathrm{r}}.x}{\hbar}}\left\langle v^{\mathrm{r}} + \frac{x}{2}\right|\hat{O}\left|v^{\mathrm{r}} - \frac{x}{2}\right\rangle$$

- Wigner function $W\left(v^{\mathrm{r}}, \pi^{\mathrm{r}}\right) = \mathscr{W}\left(\frac{\hat{\rho}_{\mathrm{r}}}{2\pi}\right) = \frac{1}{2\pi\hbar}\int \mathrm{d}x\, e^{-i\frac{\pi^{\mathrm{r}}.x}{\hbar}}\left\langle v^{\mathrm{r}} + \frac{x}{2}\right|\hat{\rho}_{\mathrm{r}}\left|v^{\mathrm{r}} - \frac{x}{2}\right\rangle$

- Satisfy for any $\hat{O}$: $\left\langle\hat{O}\right\rangle = \mathrm{tr}\left(\hat{\rho}_{\mathrm{r}}\hat{O}\right) = \int \mathrm{d}v^{\mathrm{r}}\,\mathrm{d}\pi^{\mathrm{r}}\,\mathscr{W}\left(\hat{O}\right)\left(v^{\mathrm{r}}, \pi^{\mathrm{r}}\right) W\left(v^{\mathrm{r}}, \pi^{\mathrm{r}}\right)$

- Need not be positive, phase-space quasi-probability distribution

Wigner function, geometrical representation - II

- Cosmological perturbations: Work in r/i variables $W(X) = W(X^{\mathrm{r}})W(X^{\mathrm{i}})$ and

$$W\left(v^{\mathrm{r}}, \pi^{\mathrm{r}}\right) = \frac{1}{2\pi\hbar}\int \mathrm{d}x\, e^{-i\frac{x\pi^{\mathrm{r}}}{\hbar}}\Psi_{\mathrm{r}}\left(v^{\mathrm{r}} + \frac{x}{2}\right)\Psi_{\mathrm{r}}^{*}\left(v^{\mathrm{r}} - \frac{x}{2}\right) = \frac{1}{\pi\hbar\sqrt{\det \gamma^{\mathrm{r}}}}e^{-\frac{X^{\mathrm{T}}(\gamma^{\mathrm{r}})^{-1}X}{\hbar}}$$

Gaussian, so positive

- State completely defined by covariance matrix $\gamma^{\mathrm{s}} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}$ where

$$\gamma_{11} = 2k\left\langle\left(\hat{v}_{\mathbf{k}}^{\mathrm{r}}\right)^2\right\rangle \qquad \gamma_{12} = \gamma_{21} = \left\langle \hat{v}_{\mathbf{k}}^{\mathrm{r}}\hat{\pi}_{\mathbf{k}}^{\mathrm{r}} + \hat{\pi}_{\mathbf{k}}^{\mathrm{r}}\hat{v}_{\mathbf{k}}^{\mathrm{r}}\right\rangle \qquad \gamma_{22} = \frac{2}{k}\left\langle\left(\hat{\pi}_{\mathbf{k}}^{\mathrm{s}}\right)^2\right\rangle$$

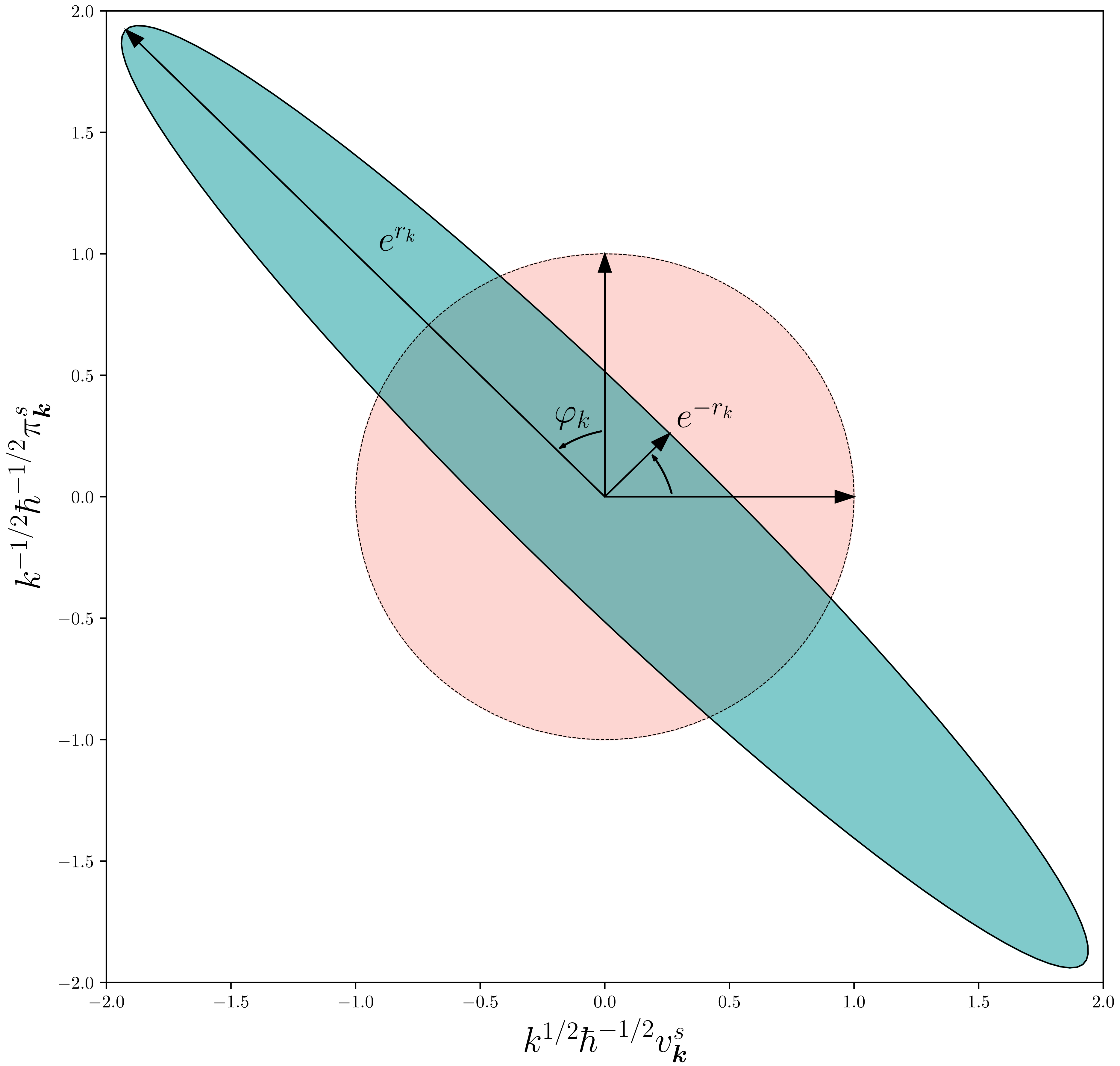
- Geometric representation: Gaussian contours levels are ellipses whose axes lengths A_k and B_k given by eigenvalues of covariance and directions by eigenbasis

- Purity $p_k = \mathrm{tr}\left(\hat{\rho}^2\right) = \dfrac{1}{\det\left(\gamma^{\mathrm{r}}\right)} = \dfrac{\hbar^2}{A_k^2 B_k^2} = \dfrac{\hbar^2\pi^2}{S_k^2}$

- Pure state: $\hat{\rho}_{\mathrm{r}} = \left|\Psi_{\mathrm{r}}\right\rangle\left\langle\Psi_{\mathrm{r}}\right|$ so $\hat{\rho}_{\mathrm{r}}^2 = \hat{\rho}_{\mathrm{r}}$ and $p_k = 1$

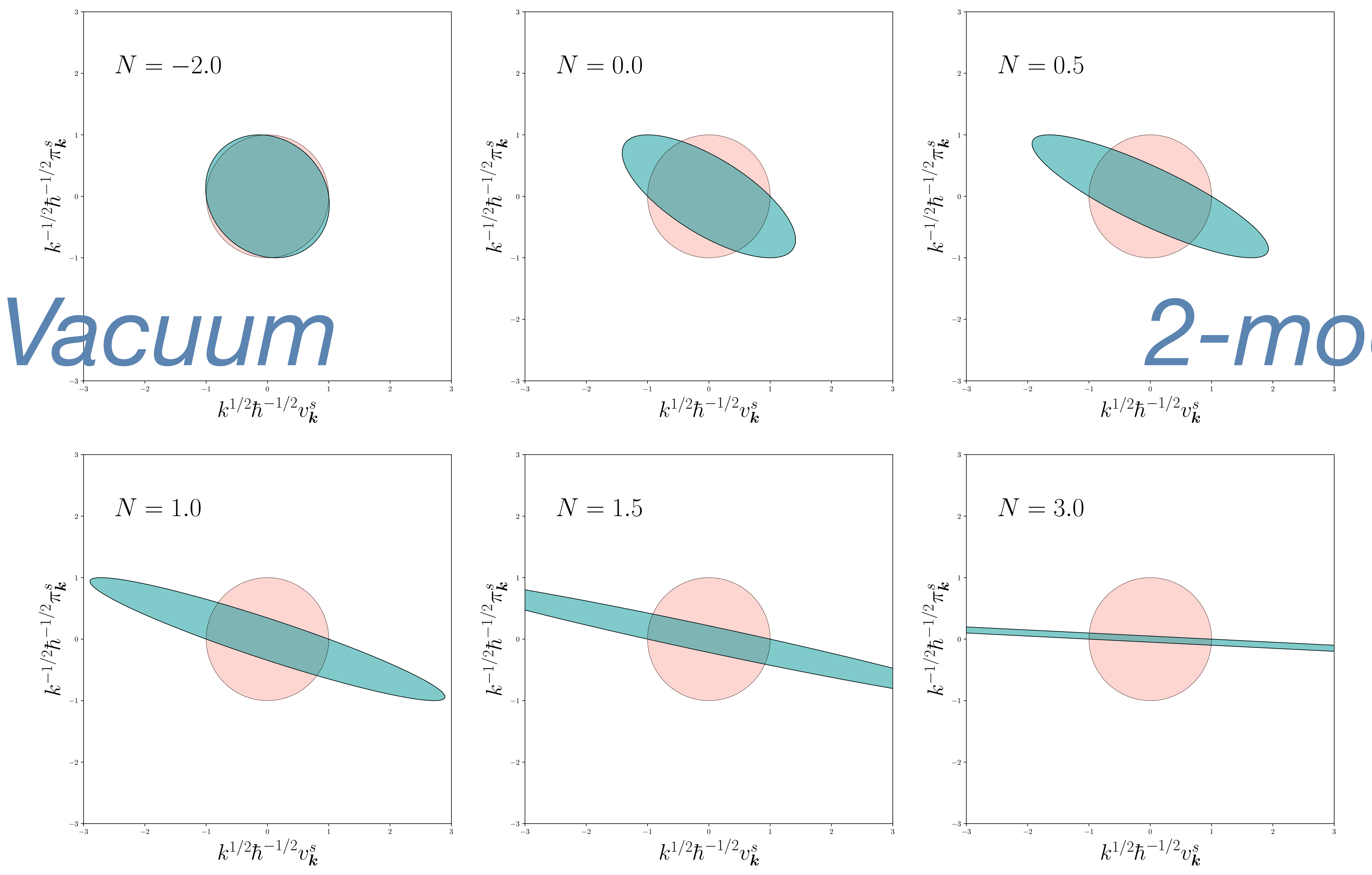
Wigner function, geometrical representation - III

- Parametrise length $e^{\pm r_k}$ by squeezing parameter r_k and direction by squeezing angle φ_k



- Covariance matrix elements

$$\gamma_{11} = \cosh\left(2r_k\right) - \cos\left(2\varphi_k\right)\sinh\left(2r_k\right)$$
$$\gamma_{12} = \gamma_{21} = -\sin\left(2\varphi_k\right)\sinh\left(2r_k\right)$$
$$\gamma_{22} = \cosh\left(2r_k\right) + \cos\left(2\varphi_k\right)\sinh\left(2r_k\right)$$



Vacuum

2-mode squeezed state

- Inflationary dynamics: very strong squeezing $r_k \approx N$.

II - Quantum signatures of the state

Apparent classicality - I

- Reminder: Perturbations described by positive Wigner function $W(X) \geq 0$

which allows to compute expectation value via $\left\langle \hat{O} \right\rangle = \int \mathrm{d}\pi^{\mathrm{r}}\, \mathrm{d}v^{\mathrm{r}}\, \mathscr{W}\left(\hat{O}\right)\left(v^{\mathrm{r}}, \pi^{\mathrm{r}}\right) W\left(v^{\mathrm{r}}, \pi^{\mathrm{r}}\right)$

How can we get anything quantum?

- Possible due to non-vanishing commutators: $\mathscr{W}\left(\hat{v}^n_{\mathbf{k}}\right) = v^n_{\mathbf{k}}$ $\mathscr{W}\left(\hat{\pi}^n_{\mathbf{k}}\right) = \pi^n_{\mathbf{k}}$ Trivial

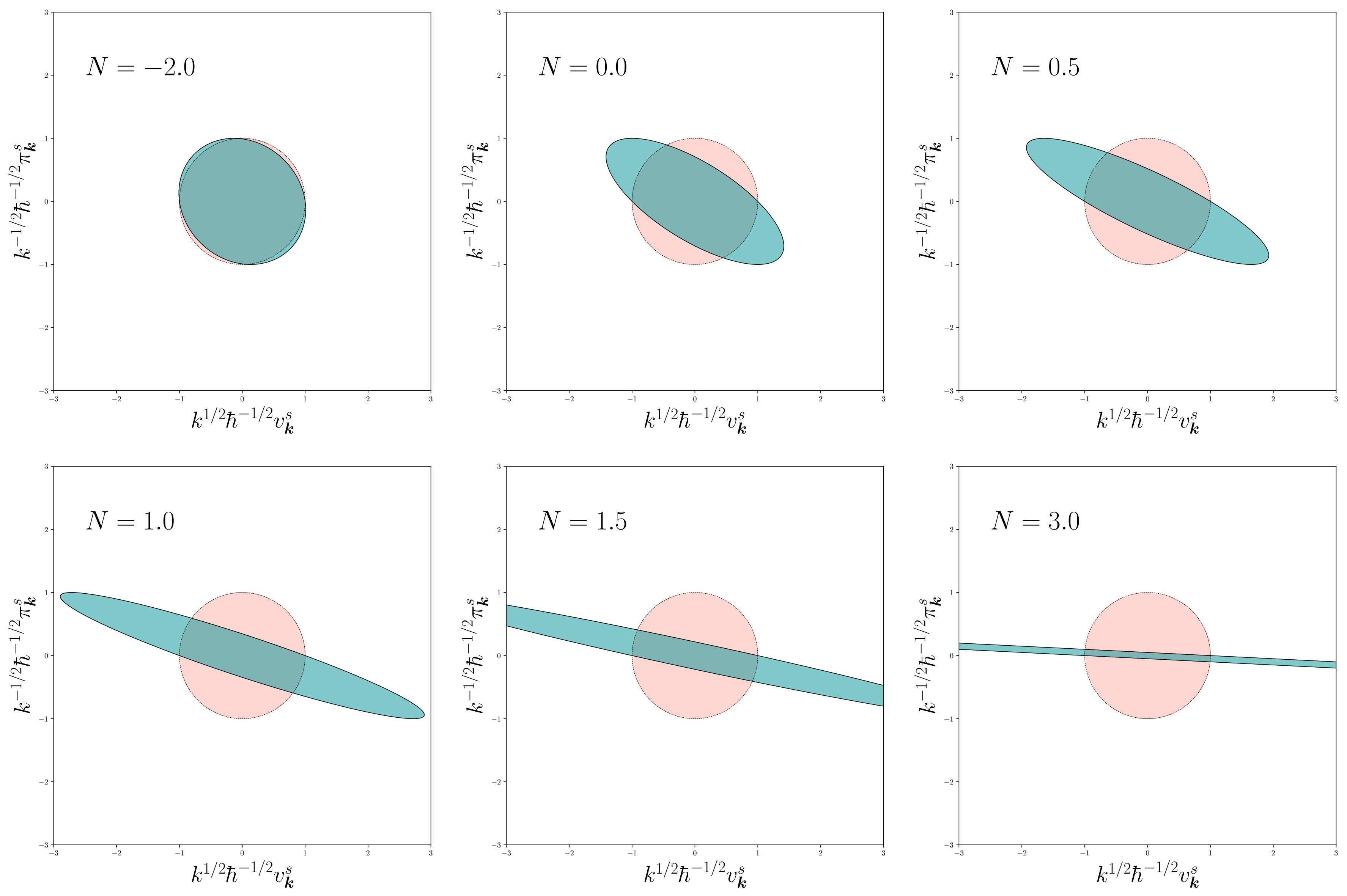
$\mathscr{W}\left(\hat{v}_{\mathbf{k}}\hat{\pi}_{\mathbf{k}} + \hat{\pi}_{\mathbf{k}}\hat{v}_{\mathbf{k}}\right) = 2v_{\mathbf{k}}\pi_{\mathbf{k}}$ Trivial: 2-point expectations values (power spectrum) described by classical probability distribution

$\mathscr{W}\left(\hat{v}^2_{\mathbf{k}}\hat{\pi}^2_{\mathbf{k}} + \hat{\pi}^2_{\mathbf{k}}\hat{v}^2_{\mathbf{k}}\right) = 2v^2_{\mathbf{k}}\pi^2_{\mathbf{k}} - \hbar$ Non-trivial, so Wigner function not strictly a probability distribution

$\mathscr{W}\left(\hat{v}^n_{\mathbf{k}}\hat{\pi}^m_{\mathbf{k}}\right) = v^n_{\mathbf{k}}\pi^m_{\mathbf{k}} + \hbar\alpha v^{n-1}_{\mathbf{k}}\pi^{m-1}_{\mathbf{k}} + \dots$ Non-trivial

When are these terms relevant?

Apparent classicality - II



- Strongly squeezed, in de Sitter limit

$$\gamma_{11} = 2k\left\langle\left(\hat{v}_{\mathbf{k}}^{\mathrm{s}}\right)^2\right\rangle \approx e^{2N}$$

$$\gamma_{12} = \gamma_{21} = \left\langle \hat{v}_{\mathbf{k}}^{\mathrm{s}}\hat{\pi}_{\mathbf{k}}^{\mathrm{s}} + \hat{\pi}_{\mathbf{k}}^{\mathrm{s}}\hat{v}_{\mathbf{k}}^{\mathrm{s}}\right\rangle \approx e^{N}$$

$$\gamma_{22} = \frac{2}{k}\left\langle\left(\hat{\pi}_{\mathbf{k}}^{\mathrm{s}}\right)^2\right\rangle \approx 1$$

$$\mathscr{W}\left(\hat{v}_{\mathbf{k}}^{n}\hat{\pi}_{\mathbf{k}}^{m}\right) = v_{\mathbf{k}}^{n}\pi_{\mathbf{k}}^{m} + \hbar\alpha v_{\mathbf{k}}^{n-1}\pi_{\mathbf{k}}^{m-1} + \ldots$$

$$\Downarrow$$

'Classical' higher-order term dominates

Take Home Message 1

As long as we consider polynomials in $\left(\hat{v}, \hat{\pi}\right)$, the expectations values at a given time can be very well reproduced by a classical probability distribution given by Wigner function.

Quantum correlations - I

- Take a closer look at the wavefunction $\Psi_{\mathbf{k},-\mathbf{k}} = \sqrt{\frac{k}{\pi\hbar\gamma_{11}}} e^{-\frac{k}{\hbar}\frac{(1-i\gamma_{12})}{\gamma_{11}} v_{\mathbf{k}} v_{-\mathbf{k}}} \neq \Psi_{\mathbf{k}}\Psi_{-\mathbf{k}}$

  Other ways to make it manifest?

  Entangled!

- Paradigm: Quantumness of a state for a system $\mathcal{S}$ = Quantumness of correlations of subsystems $\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2$ for this state.

Goal: Show that correlations are stronger than classically allowed e.g. Bell inequality

- Another instance is the Quantum Discord $\mathcal{D}\left(\mathcal{S}_1, \mathcal{S}_2\right)$

$$\mathcal{D}\left(\mathcal{S}_1, \mathcal{S}_2\right) \equiv I(\mathcal{S}_1, \mathcal{S}_2) - \max_{\{\hat{\Pi}_j^{\mathcal{S}_2}\}} J\left(\mathcal{S}_1, \mathcal{S}_2, \{\hat{\Pi}_j^{\mathcal{S}_2}\}\right)$$

  with I, J two measures of mutual information between $\mathcal{S}_{1/2}$.

If $\mathcal{S}_i$ described by classical probabilities $\mathcal{D}\left(\mathcal{S}_1, \mathcal{S}_2\right) = 0$.

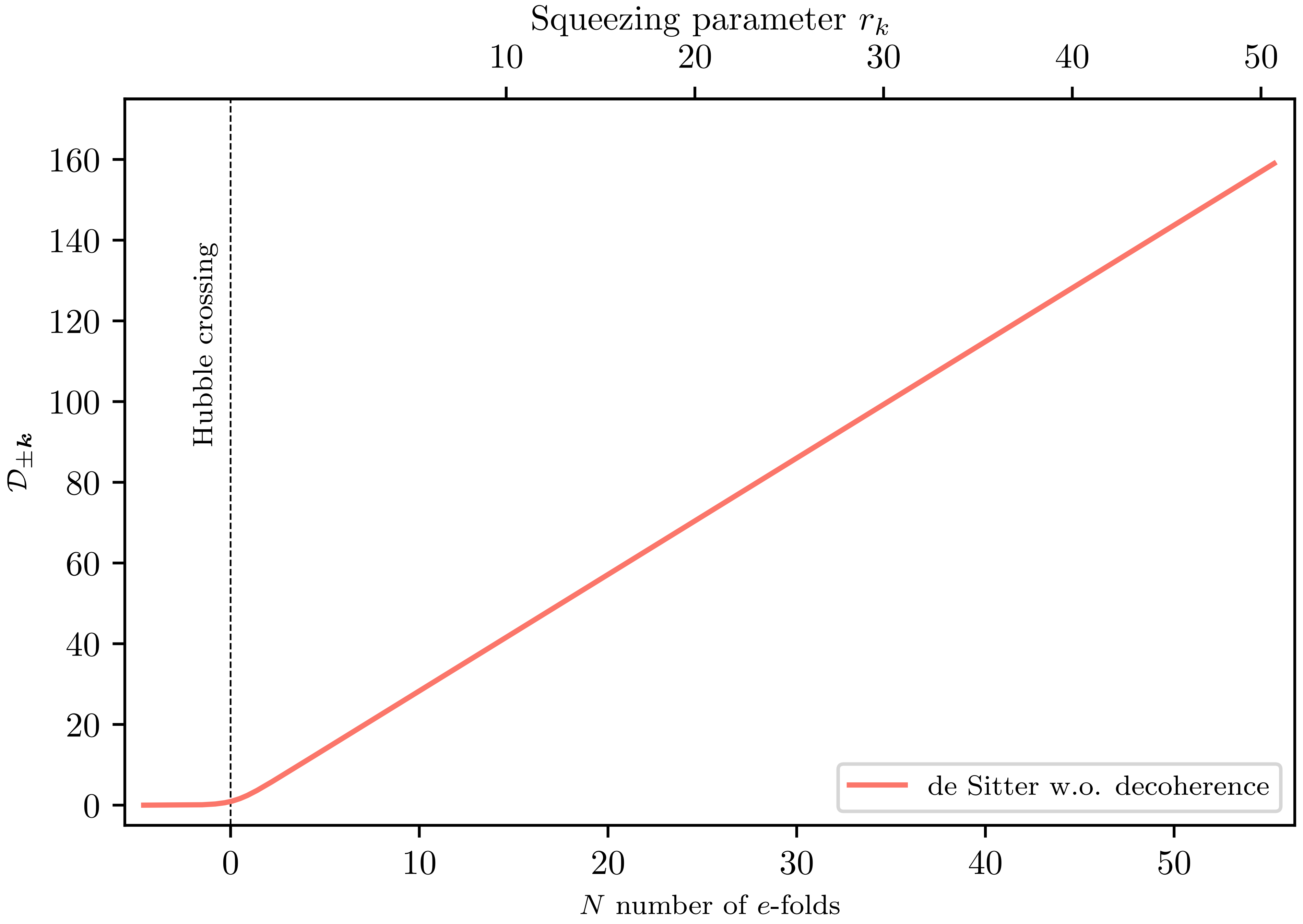
Quantum setting $\mathcal{D}\left(\mathcal{S}_1, \mathcal{S}_2\right) \geq 0$.

Quantum correlations - II

- Subsystems? At quadratic order perturbations are in independent $\pm\mathbf{k}$ pairs
  → Consider one of the pairs.
- Gaussian state: Discord can be computed using covariance matrix elements directly[2]

$$\mathscr{D}_{\pm\mathbf{k}} = f\left[\cosh\left(2r_k\right)\right] \quad \text{with} \quad f(x) = \left(\frac{x+1}{2}\right)\log_2\left(\frac{x+1}{2}\right) - \left(\frac{x-1}{2}\right)\log_2\left(\frac{x-1}{2}\right)$$

- For large squeezing $\mathscr{D}_{\pm\mathbf{k}} \approx 2r_k/\ln 2 \approx 2N/\ln 2$
- The state also violates a Bell inequality[2]



Take Home Message 2

Dynamics generates strong quantum correlations between $\pm k$ modes in the sense of several non-classicality criteria.

2. [arXiv:2211.10114 Martin, Micheli, and Vennin]

Quantum correlations - III

- *Is there a contradiction?* No!

$$\left\langle \hat{O} \right\rangle = \int \mathrm{d}\pi^{\mathrm{r}}\, \mathrm{d}v^{\mathrm{r}}\, \mathscr{W}\left(\hat{O}\right)\left(v^{\mathrm{r}}, \pi^{\mathrm{r}}\right) W\left(v^{\mathrm{r}}, \pi^{\mathrm{r}}\right)$$

Need $\mathscr{W}\left(\hat{O}\right)$ non-polynomial in (v, π) to be revealed.

Take Home Message 3

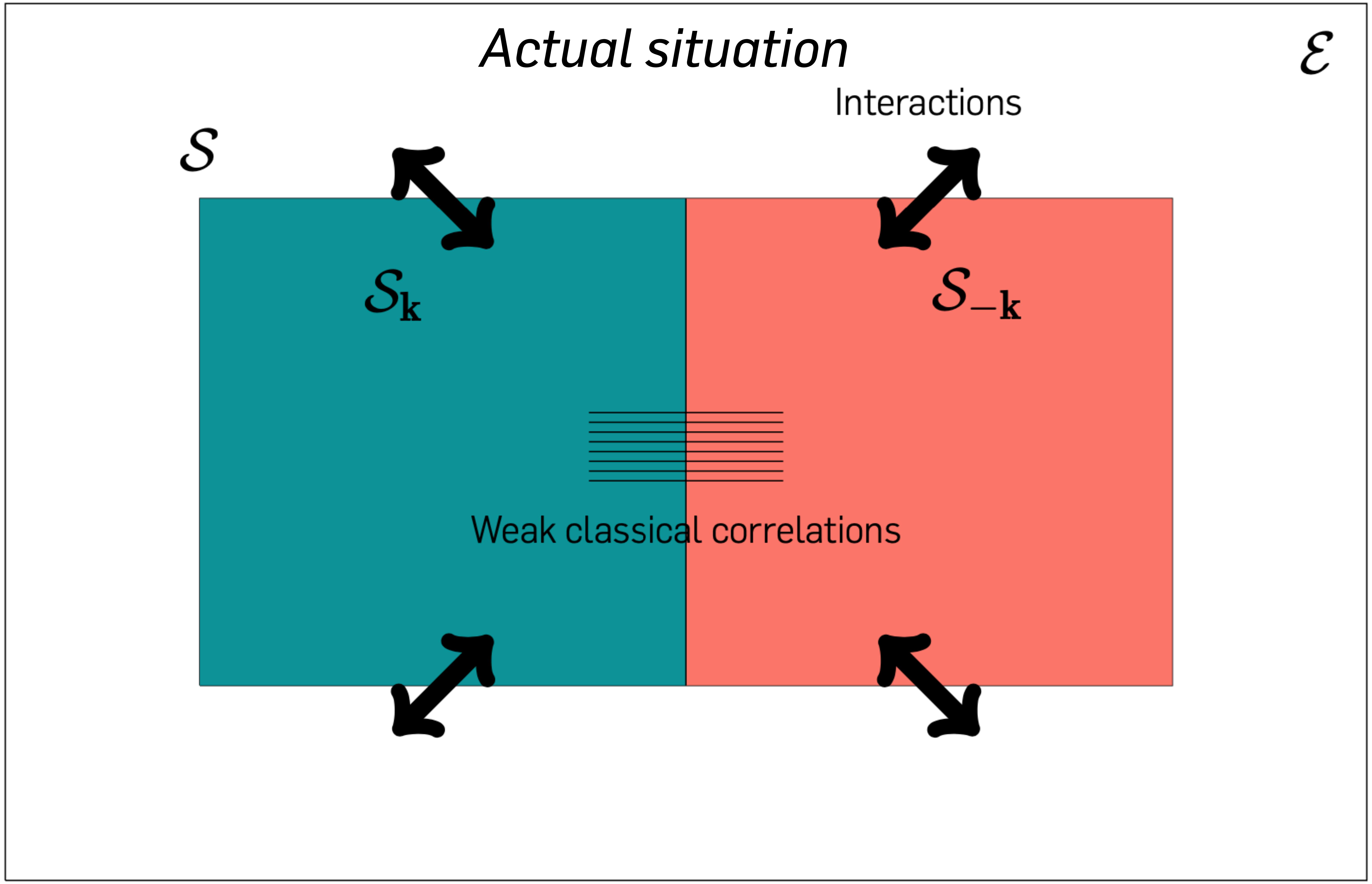
Quantum correlations *are* present but, unfortunately, *manifest* only for a class of observables which are not the ones routinely measured by cosmologists.

- Example: One such operator is $\hat{\sigma}_z = -\int_{-\infty}^{\infty} \left| v_{\mathbf{k}} \right\rangle \left\langle -v_{\mathbf{k}} \right| \mathrm{d}v_{\mathbf{k}}$ such that $\hat{\sigma}_z^2 = 1$, eigenvalues ± 1
- Its Weyl transform reads: $\mathscr{W}\left(\hat{\sigma}_z\right) = -\pi\delta\left(v_{\mathbf{k}}\right)\delta\left(\pi_{\mathbf{k}}\right)$ Non-analytical in (v, π)

In principle, quantum correlations in the state,
but are they robust against interactions?

III - Decoherence: Destruction of quantum correlations

Decoherence : how to destroy quantum features



Interactions with extra d.o.f lead to decoherence of quantum systems.

Environment destroys quantum correlations

- $\mathcal{S}$ = pair of cosmological perturbations modes $\pm\mathbf{k}$.
- $\mathcal{E}$ = other $\pm\mathbf{k}'$ pairs and other fields.
- Model: $\hat{H}_{\text{int}}(\tau) = g\int \mathrm{d}^3\mathbf{x}\,\hat{v}(\mathbf{x})\otimes\hat{E}(\tau,\mathbf{x})$

Interaction taken linear to preserve Gaussianity, independence of $\pm\mathbf{k}$ pairs

$$\hat{\rho}_{\mathcal{S}} = \bigotimes_{\mathbf{k}\in\mathbb{R}^{3},+}\hat{\rho}_{\pm\mathbf{k}}$$

- Under a few generic assumptions (perturbative coupling, $\mathcal{E}$ large w.r.t $\mathcal{S}$ and stationnary) can derive Lindblad equation (non-unitary)

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}\tau} = -i\left[\hat{H},\hat{\rho}\right] - \frac{\Gamma}{2}\int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{y}\,C_E(\tau;\mathbf{x},\mathbf{y})\left[\hat{v}(\mathbf{x}),\left[\hat{v}(\mathbf{y}),\hat{\rho}\right]\right]$$

with

$$C_E(\tau;\mathbf{x},\mathbf{y}) = \langle\hat{E}(\tau,\mathbf{x})\hat{E}(\tau,\mathbf{y})\rangle$$

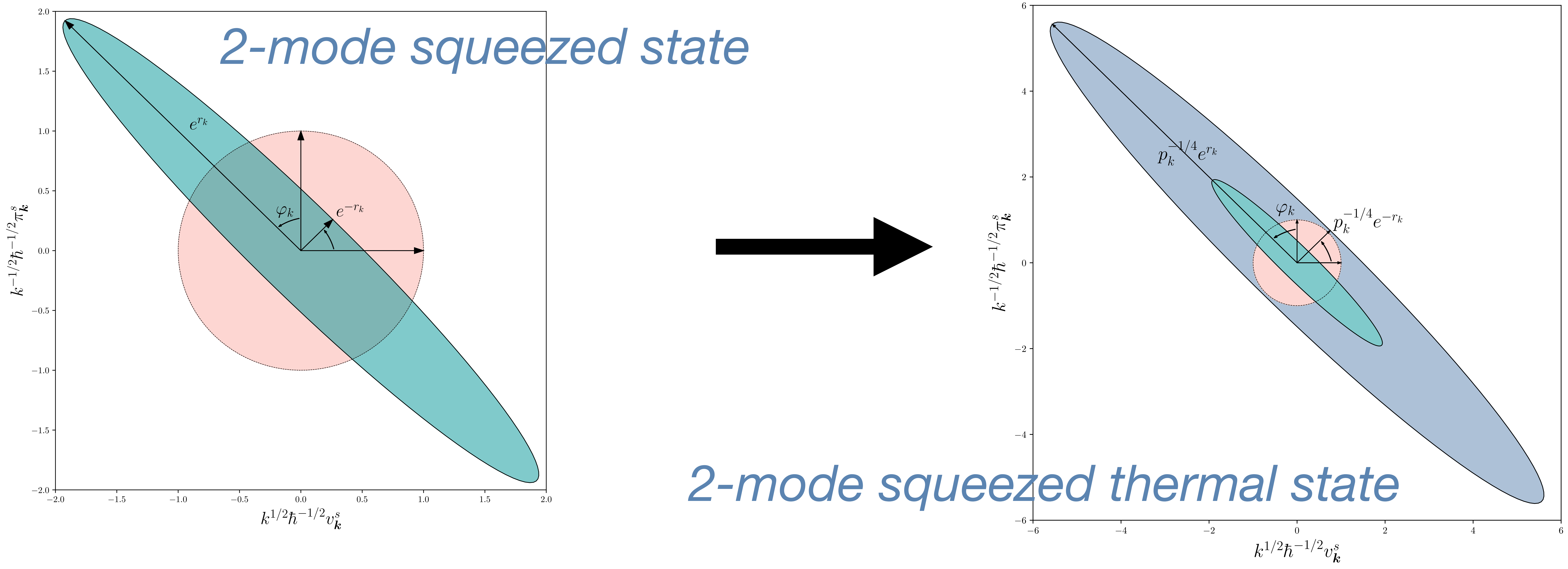
$$\Gamma = 2g^2\tau_c$$

- $\hat{\rho}_{\pm\mathbf{k}}$ becomes mixed 2-mode squeezed state[3] parametrised by r_k, φ_k and the purity $0\leq p_k\leq 1$.

3. [arXiv:2112.05037 Martin, Micheli and Vennin]

Environment destroys quantum correlations

- Geometrically: growth of the ellipse area $S_k = \pi\hbar/\sqrt{p_k}$



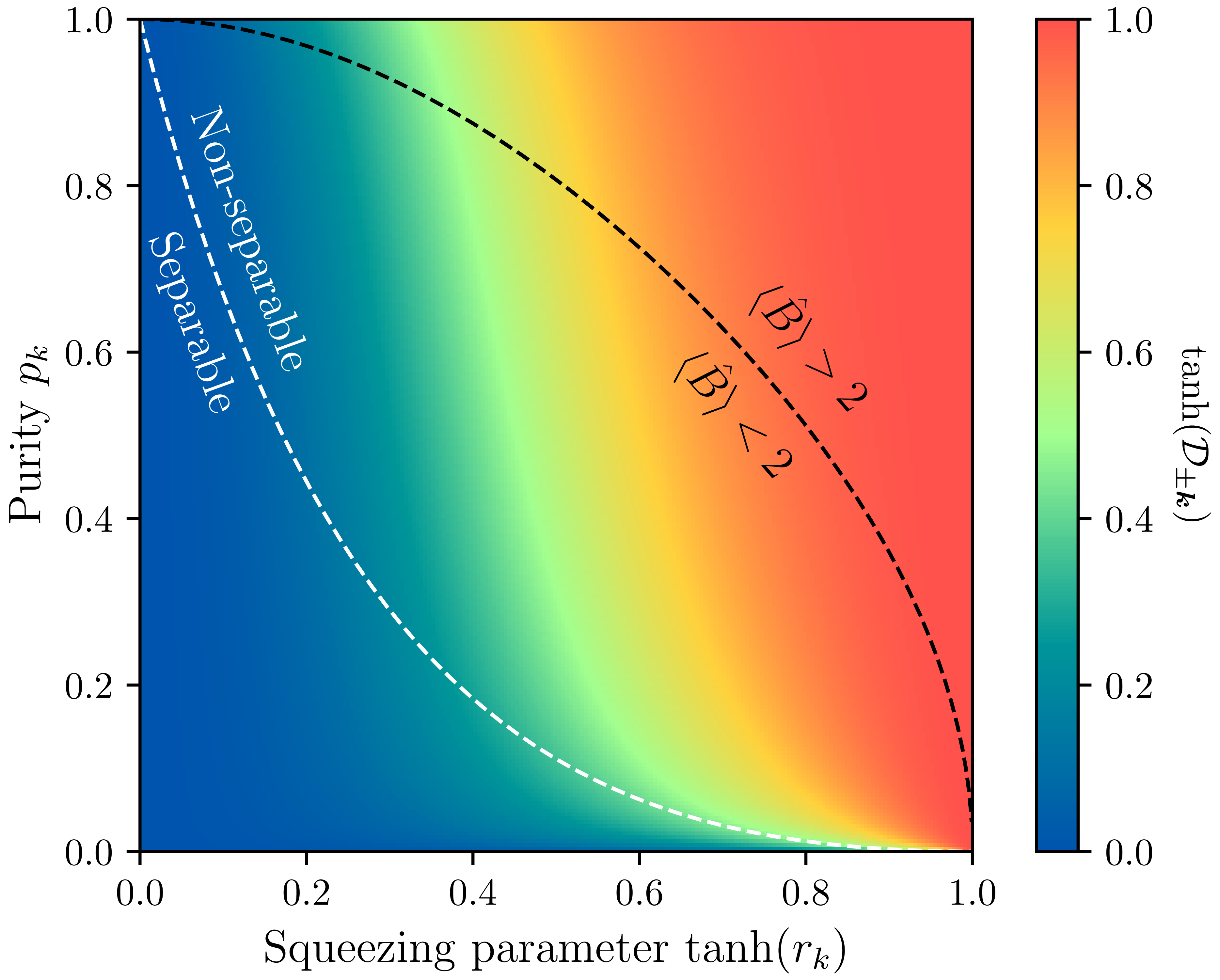
- Covariance matrix elements multiplied by same overall factor of $p_k^{-1/2}$ e.g.

$$\gamma_{11} = p_k^{-1/2}\left[\cosh\left(2r_k\right) - \cos\left(2\varphi_k\right)\sinh\left(2r_k\right)\right]$$

How does decoherence affect quantumness of correlations?

Environment destroys quantum correlations

- Discord in presence of decoherence[2,3] $\mathscr{D}_{\pm\mathbf{k}} = f\left[p_k^{-1/2}\cosh\left(2r_k\right)\right] - 2f\left(p_k^{-1/2}\right) + f\left[\frac{p_k^{-1/2}\cosh\left(2r_k\right) + p_k^{-1}}{p_k^{-1/2}\cosh\left(2r_k\right) + 1}\right]$



Take Home Message 4

Quantum correlations can always be erased by sufficient decoherence but there is a competition between correlation build up and interaction erasing quantum features.[3,5]

Where are we in this plot for the precise dynamics of inflation?

p_k recently computed in [4] for non-linearities of pure gravity, would need to compare!

2. [arXiv:2211.10114 Martin, Micheli, and Vennin]
3. [arXiv:2112.05037 Martin, Micheli and Vennin]
4. [arXiv:2211.11046 Burgess et al.]

Conclusions and future directions

Conclusions

- Without decoherence the state of the perturbations undoubtedly exhibit quantum correlations but…

A. Latest results on the level of decoherence tend to show that perturbations classicalised on cosmological scales.

B. In any case, no proposed protocol to measure these criteria; two reasons why:

   1. Only measure curvature $\hat{v}(x) \sim \hat{\zeta}(x)$ at a single time: miss another non-commuting observable $\hat{\pi}(x)$.

   2. Even measuring $\hat{\pi}(x) \sim \partial_t \hat{\zeta}(x)$ would not be sufficient as explained, would need to measure a complicated combination of both.

Future directions

- If we want to go further need either to have several times, or go beyond Gaussian level or consider more complicated models.

C. Could try to exploit *unequal time* correlations i.e. tomography $\hat{\zeta}\left(x, t_1\right)$ and $\hat{\zeta}\left(x, t_2\right)$, using temporal Bell inequalities.

D. Non-gaussian signal + certain assumptions (à la Green and Porto[5])

E. Beyond single-field slow roll (à la Maldacena[6]) where other fields could store quantumness robustly.

5. [arXiv:2001.09149 Green and Porto]
6. [arXiv:1508.01082 Maldacena]

Thank you for your attention!

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